

## FULLWAVE DESCRIPTION OF PROPAGATION AND LOSSES IN QUASIPLANAR TRANSMISSION LINES BY QUASI-ANALYTICAL SOLUTION

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### Abstract

In this paper, we present a fullwave description of propagation and losses for some quasiplanar transmission lines by using a quasi-analytical solution. This latter is derived from a recently proposed modified transverse resonance method (MTRM), in which an analytical preprocessing has been introduced. The quasistatic contribution is obtained by an *entirely analytical solution*, so the resultant system of linear equations is very efficient. Furthermore, the *resistive boundary conditions* as well as the *complex substrat permittivity* are taken into account in an intrinsic manner, leading to an accurate determination of dielectric and conductor losses in lossy transmission lines. Theoretical and experimental results will be presented respectively for a lossless CPW and a lossy microstrip line.

### Introduction

Planar and quasiplanar transmission lines have been subject to a large number of studies during last three decades, by both quasistatic and fullwave methods. The most representative has been collected in the books edited by T. Itoh [1] and by R. Sorrentino [2]. Several recent publications have been focused on the derivation of very fast, then analytical or quasi-analytical formulations. F. Medina and M. Horno have studied the case of boxed microstrip line in layered medium [3], and S.S. Bedair and I. Wolff have been interested in supported coplanar waveguide [4]. Due to an analytical preprocessing, drastic improvement of accuracy and the CPU time has been achieved by Medina and Horno in [3]. This makes the formulation suitable for microwave and millimeter-wave CAD purpose. It will be nevertheless noted that both [3] and [4] deal with quasistatic cases ; moreover, the dielectric and

conductor losses have not been considered.

The fullwave characterization we propose is suited for both propagation and losses for some commonly used quasiplanar transmission lines, by using the modified transverse resonance method (MTRM) [5]. An analytical preprocessing is introduced, allowing the extraction of the static contribution. The resolution of resultant linear equations system is very fast since the matrix elements are of quasi-analytical type. An analysis program has been developed in FORTRAN on a PC computer. The theoretical results for a lossless supported coplanar waveguide (SCPW) and a lossy microstrip line are in good agreement with either previously published results or experiments made in our Laboratory.

### Formulation

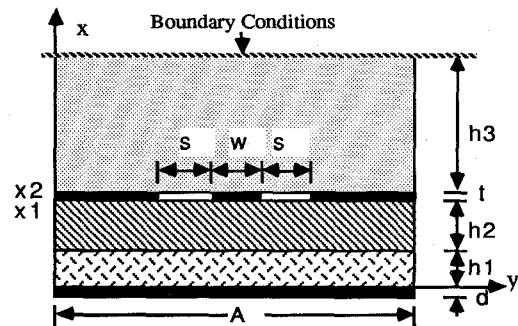


Fig.1 General configuration studied by proposed formulation

By considering the structure of Fig.1, the modified transverse resonance method (MTRM) is applied as follows :

- 1) the external surface current densities  $J_s(x_1-\Delta, y)$  and  $J_s(x_2+\Delta, y)$  are related to the tangential electric fields  $E_t(x_1, y)$

and  $E_t(x_2, y)$  at  $h_1 + h_2$  and  $h_1 + h_2 + t$  via associated susceptance operators [5]. Any kind of boundary conditions (electric or magnetic wall, open condition or complex resistive boundary condition for finite conductivity metallization), as well as the complex permittivity, are taken into account in the formulation of susceptance operator concerned ;

2) the internal surface current densities  $J_s(x_1 + \Delta, y)$ ,  $J_s(x_2 - \Delta, y)$  are functions of  $E_t(x_1, y)$  and  $E_t(x_2, y)$ , by making use of the transmission matrix of the parallel-plate waveguide (PPW) model in the metallization region ( $x_1 < x < x_2$ ) ;

3) the continuity of surface current densities in the aperture regions ( $0.5(A-w-2s) < y < 0.5(A-w)$ ,  $0.5(A+w) < y < 0.5(A+w+2s)$ ) at interfaces  $x_1$  and  $x_2$  leads to an integral equation with  $E_t(x_1, y)$

and  $E_t(x_2, y)$  as unknown ;

4) the application of the Ritz-Galerkin method by developing  $E_t(x_1, y)$  and  $E_t(x_2, y)$  in Chebychev polynomials of first and second kind with appropriate edge condition leads to a system of linear equations for which a non trivial solution allows a complete fullwave characterization of the structure of Fig.1.

Due to their advantages, Chebychev polynomials has been adopted as trial function by many authors [1-3,6]. These advantages are : first, their Fourier transform are analytical (Bessel functions), then no numerical integration is needed ; second, accurate results can be achieved by using only few Chebychev polynomials, the system of linear equations is then of very moderate size.

Inspite of these advantages, the computation efficiency is not satisfactory because of the infinite summation of Fourier type in each matrix element. Indeed, due to the static contribution, as mentionned in [3], the convergence is very poor for the concerned series, and 4000 terms are used in the alternative formulation of [4].

By extracting from these series the quasistatic part which is independant of frequency, we obtain a very quickly convergent serie which can be truncated after about ten terms.

The quasistatic part being evaluated just once for each structure, the resultant system is very efficient.

This quasistatic solution is obtained by tending the frequency and the propagation constant towards zero. Because of the Bessel functions involved, this serie can only be evaluated numerically, as in a similar formulation for boxed microstrip [6].

In our case, we have obtained an *entirely analytical solution* by replacing the Bessel function by it's large argument approximation with  $(k.r)^{-1/2}$  and  $(k.r)^{-3/2}$  terms. The difference becomes noticeable only for arguments less than the first root of  $J_0(k.r)$ , which corresponds generally to the first three terms of the original serie in most commonly used microstrip line case. Furthermore, better accuracy can be achieved since no truncature is needed for the static part of each matrix element.

An analysis program has been developed on a PC computer. When considering the cases of no losses (dielectric, conductor, leakage) nor complex modes, an automatic scanning procedure on the entire  $Re(\gamma^2)$  axis is used. This procedure consists in : 1) determining the poles of characteristics equation ; 2) researching the existance of one or two roots between two poles by using a quadratic procedure, and determining for each root the sub-interval for which the characteristic equation varies quasi-linearly ; 3) using the classical secant method to determine accurately the roots of characteristic equation.

This automatic scanning procedure can be very useful for determining the whole mode spectrum when characterizing uniaxial discontinuities in quasiplanar transmission lines by the Mode-Matching method. For complex spectra, this procedure allows a fast localization of these spectra by observing the modes disappearance phenomena ; a scanning in the complex  $\gamma$  plane can then be used.

## Results

Two examples are given here to illustrate the efficiency of the proposed solution. In both case, the effective dielectric constant and the characteristic impedance are obtained by using different values of  $M$ , the number of trial functions which determine the matrix size, and  $N$ , the truncation number of the

Table I  
Convergence of  $\epsilon_{\text{eff}}$  and  $Z_0$  for SCPW at 1GHz  
 $s=200\mu\text{m}$ ,  $w=120\mu\text{m}$ ,  $h_1=h_3=10\text{mm}$ ,  
 $h_2=200\mu\text{m}$ ,  $\epsilon_{r1}=3.78$ ,  $\epsilon_{r2}=12.9$ ,  $\epsilon_{r3}=1$   
Results of [4] with 4000 spectral terms are :  
 $\epsilon_{\text{eff}}=6.2932$ ,  $Z_0=67.97$

M	N	5	10	15	20	25	30	40	50
2	$\epsilon_{\text{eff}}$	6.6878	6.4165	6.3169	6.2948	6.2915	6.2912	6.2912	6.2912
	$Z_0$	68.669	70.102	70.652	70.776	70.794	70.796	70.796	70.796
3	$\epsilon_{\text{eff}}$	6.6879	6.4175	6.3186	6.2969	6.2937	6.2934	6.2934	6.2934
	$Z_0$	67.234	68.676	69.250	69.388	69.410	69.412	69.412	69.412
4	$\epsilon_{\text{eff}}$	6.6879	6.4175	6.3186	6.2968	6.2937	6.2934	6.2933	6.2933
	$Z_0$	67.112	68.551	69.125	69.262	69.284	69.286	69.286	69.286

Table II  
Convergence of  $\epsilon_{\text{eff}}$  and  $Z_0$  for boxed microstrip at  $h_2/\Delta_Q = 0.01$   
 $w=1\text{mm}$ ,  $h_1=0$ ,  $h_2=1\text{mm}$ ,  $h_3=9\text{mm}$   
 $\epsilon_{r1}=1$ ,  $\epsilon_{r2}=12.5$ ,  $\epsilon_{r3}=1$

M	N	$t=0$				$t=1\mu\text{m}$			
		5	10	15	20	5	10	15	20
3	$\epsilon_{\text{eff}}$	8.4734	8.4852	8.4850	8.4849	8.4321	8.4695	8.4691	8.4690
	$Z_0$	50.709	50.309	50.294	50.292	50.264	49.499	49.468	49.467
4	$\epsilon_{\text{eff}}$	8.4777	8.4500	8.4478	8.4477	8.4707	8.4354	8.4309	8.4308
	$Z_0$	51.724	48.404	48.260	48.250	53.827	47.850	47.560	47.553
5	$\epsilon_{\text{eff}}$	8.4161	8.4501	8.4480	8.4478	8.2857	8.4358	8.4313	8.4312
	$Z_0$	56.538	49.045	48.487	48.455	63.931	48.934	47.882	47.859

modified serie. The results are given respectively in Table I for a SCPW, and in Table II for a boxed microstrip line with zero and  $1\mu\text{m}$  strip thickness.

We can see that the matrix elements converge more quickly for microstrip line than for SCPW, since only 15 serie terms are needed for microstrip line to achieve a five digit accuracy for both  $\epsilon_{\text{eff}}$  and  $Z_0$ , when 25 terms are necessary for SCPW.

This is related to the small aspect ratio ( $r = s/A$ ) of SCPW.

In contrast, to achieve an accuracy of five digits for  $\epsilon_{\text{eff}}$  and three digits for  $Z_0$ , only 3 trial functions are sufficient for

SCPW, which results a 5x5 matrix equation, when 4 trial functions are needed for microstrip lines.

By using 10 series terms and 4 trial functions in the automatic scanning procedure, the computation of 10 modes (propagating and evanescent) in a boxed microstrip at 100 frequencies takes about 5 minutes on an Intel-486 based PC computer .

Dielectric and ground plane conductor losses have also been studied in a boxed microstrip line by varying the dielectric loss tangents and the ground plane (considered as bulky) conductivity. The results of propagation constant and normalized losses are given in Fig.2, compared to those of [7] for an open microstrip line. The results are very close each other for the small losses case, also for large dielectric losses

case. Our results for no loss propagation constant, dielectric losses with  $\tan\delta=10^{-3}$ , and ground plane losses with  $\sigma=\sigma_{\text{cu}}$  are respectively  $0.8662\text{ cm}^{-1}$ ,  $3.789\text{E-}4\text{ cm}^{-1}$  and  $1.125\text{E-}4\text{ cm}^{-1}$ , while those given in [7] are  $0.866\text{ cm}^{-1}$ ,  $3.766\text{E-}4\text{ cm}^{-1}$  and  $1.120\text{E-}4\text{ cm}^{-1}$ . The difference of ground plane conductor losses becomes important in these two structures for moderate conductivities. All results presented here clearly illustrate the limit of the classical perturbation method in lossy transmission line studies.

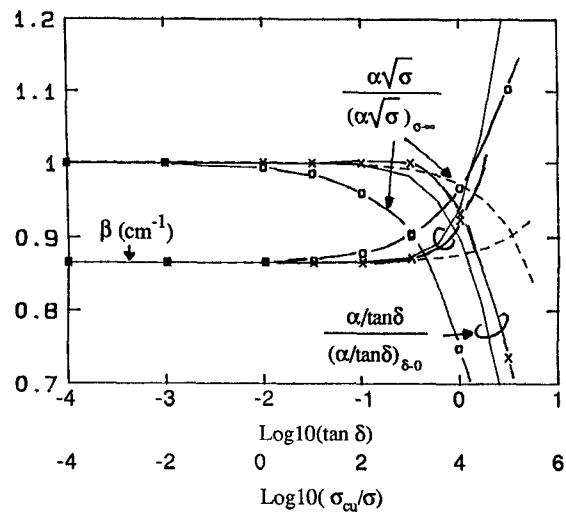


Fig.2 Ground plane conductor losses (---) and dielectric losses (—) of a boxed microstrip line at 3GHz ; Comparison with results taken from [7] ( o, x) for an open microstrip line.  
 $w=5\text{mm}$ ,  $h_1=0$ ,  $h_2=1.6\text{mm}$ ,  $h_3=20\text{mm}$ ,  $\epsilon_{r1}=1$ ,  $\epsilon_{r2}=2.2$ ,  $\epsilon_{r3}=1$ ,  $A=50\text{mm}$

Finally, a lossy microstrip line has been studied by using the proposed formulation and also a perturbation formulation. The thickness of the strip and of the ground plane are the same, and comparable to the skin depth in the working frequency range. The theoretical results of effective dielectric constant and losses are shown in Fig.3, and compared to measurements carried out in our Laboratory on a vector network analyzer (VNA). Better agreement has been obtained by this formulation than the perturbation analysis, as can be waited in a moderately lossy structure.

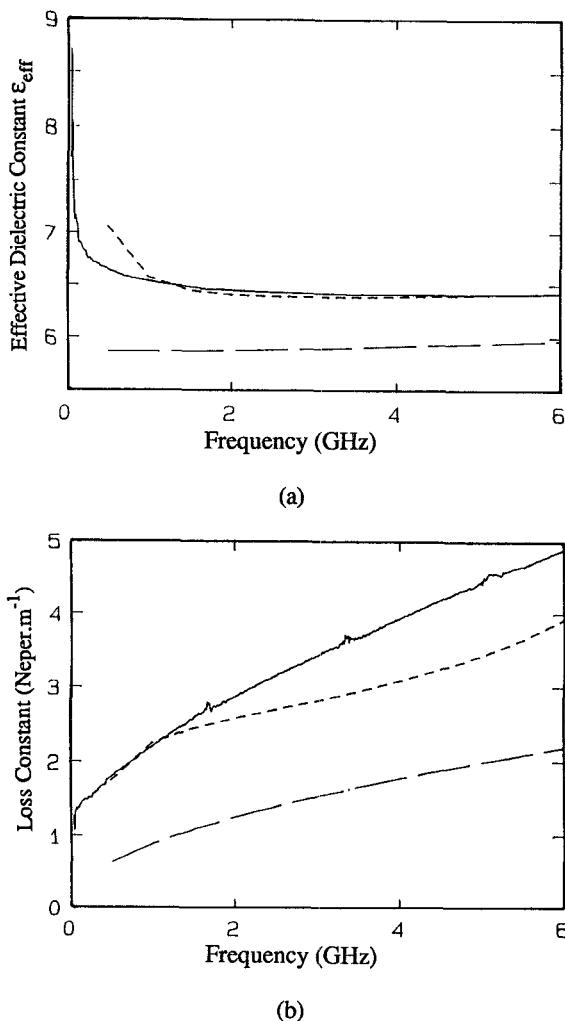


Fig.3 Comparison between measurement (—), this formulation (---) and perturbation theory (....) for a lossy microstrip line.

(a) effective dielectric constant; (b) conductors losses ;  
 $w=210\mu\text{m}$ ,  $h_1=0$ ,  $h_2=635\mu\text{m}$ ,  $h_3=10\text{mm}$ ,  
 $\epsilon_{r1}=1$ ,  $\epsilon_{r2}=9.8$ ,  $\epsilon_{r3}=1$ ,  $t=d=10\mu\text{m}$ ,  $\sigma=8E5 \text{ S/m}$ .

## Conclusion

A quasi-analytical formulation has been presented for characterizing some commonly used quasiplanar transmission lines. This formulation is derived from a fullwave method, namely, the modified transverse resonance method (MTRM). It has been applied with success to both coplanar waveguide and boxed microstrip line. The numerical efficiency of this method allows the computation of mode spectra, propagating and evanescent, in a quasiplanar structure in very short CPU time on a PC computer, making this method well suited to discontinuity analysis and other CAD purposes.

## References

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